



General Certificate of Education

Mathematics 6360

MD02 Discrete 2

Mark Scheme

2005 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous		
	incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

Application of Mark Scheme

No method shown:

Correct answer without working
Incorrect answer without working

mark as in scheme
zero marks unless specified otherwise

More than one method / choice of solution:

2 or more complete attempts, neither/none crossed out

mark both/all fully and award the mean
mark rounded down

1 complete and 1 partial attempt, neither crossed out

award credit for the complete solution only

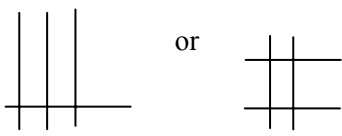
Crossed out work

do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method

award method and accuracy marks as
appropriate

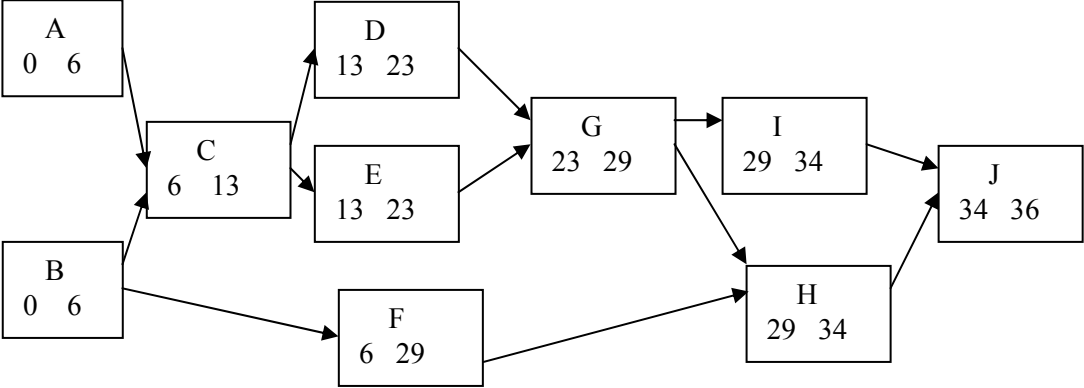
MD02

Q	Solution	Marks	Total	Comments																																																																																																																														
1(a)	Hungarian algorithm minimises 20-x gives measure of questions not correct which needs minimising	E1 E1	2																																																																																																																															
(b)	<table style="border-collapse: collapse; margin-bottom: 10px;"> <tr><td>3</td><td>1</td><td>2</td><td>5</td><td>4</td></tr> <tr><td>0</td><td>2</td><td>5</td><td>1</td><td>3</td></tr> <tr><td>7</td><td>3</td><td>3</td><td>4</td><td>6</td></tr> <tr><td>8</td><td>4</td><td>2</td><td>5</td><td>6</td></tr> <tr><td>6</td><td>4</td><td>5</td><td>4</td><td>5</td></tr> </table> <table style="border-collapse: collapse; margin-bottom: 10px;"> <tr><td>2</td><td>0</td><td>1</td><td>4</td><td>3</td></tr> <tr><td>0</td><td>2</td><td>5</td><td>1</td><td>3</td></tr> <tr><td>4</td><td>0</td><td>0</td><td>1</td><td>3</td></tr> <tr><td>6</td><td>2</td><td>0</td><td>3</td><td>4</td></tr> <tr><td>2</td><td>0</td><td>1</td><td>0</td><td>1</td></tr> </table> <table style="border-collapse: collapse; margin-bottom: 10px;"> <tr><td>2</td><td>0</td><td>1</td><td>4</td><td>2</td></tr> <tr><td>0</td><td>2</td><td>5</td><td>1</td><td>2</td></tr> <tr><td>4</td><td>0</td><td>0</td><td>1</td><td>2</td></tr> <tr><td>6</td><td>2</td><td>0</td><td>3</td><td>3</td></tr> <tr><td>2</td><td>0</td><td>1</td><td>0</td><td>0</td></tr> </table> <p>Zeros can be covered with only 4 lines So adjustment by 1</p> <table style="border-collapse: collapse; margin-bottom: 10px;"> <tr><td>2</td><td>0</td><td>1</td><td>3</td><td>1</td></tr> <tr><td>0</td><td>2</td><td>5</td><td>0</td><td>1</td></tr> <tr><td>4</td><td>0</td><td>0</td><td>0</td><td>1</td></tr> <tr><td>6</td><td>2</td><td>0</td><td>2</td><td>2</td></tr> <tr><td>3</td><td>1</td><td>2</td><td>0</td><td>0</td></tr> </table> <p>Matching on particular zeros</p> <p>Les- Tennis Mel - Athletics Nick- Swimming Ollie- Football Pete- Golf</p>	3	1	2	5	4	0	2	5	1	3	7	3	3	4	6	8	4	2	5	6	6	4	5	4	5	2	0	1	4	3	0	2	5	1	3	4	0	0	1	3	6	2	0	3	4	2	0	1	0	1	2	0	1	4	2	0	2	5	1	2	4	0	0	1	2	6	2	0	3	3	2	0	1	0	0	2	0	1	3	1	0	2	5	0	1	4	0	0	0	1	6	2	0	2	2	3	1	2	0	0	<p>B1</p> <p>M1</p> <p>A1✓</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p>	<p>Array giving 20-x</p> <p>Reduce rows</p> <p>ft their 20 - x matrix</p> <p>Reduce columns</p> <p>CSO</p> <p>  </p> <p>or</p> <table style="border-collapse: collapse; margin-left: 40px;"> <tr><td>1</td><td>0</td><td>1</td><td>3</td><td>1</td></tr> <tr><td>0</td><td>3</td><td>6</td><td>1</td><td>2</td></tr> <tr><td>3</td><td>0</td><td>0</td><td>0</td><td>1</td></tr> <tr><td>5</td><td>2</td><td>0</td><td>2</td><td>2</td></tr> <tr><td>2</td><td>1</td><td>2</td><td>0</td><td>0</td></tr> </table> <p>If adjustment not done correctly and selection is made 3 correct matchings B1 rest correct B1</p> <p>Award marks here in whichever way benefits candidate most.</p>	1	0	1	3	1	0	3	6	1	2	3	0	0	0	1	5	2	0	2	2	2	1	2	0	0	<p>A1</p> <p>A1</p>	<p>9</p>
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Total			11																																																																																																																															

MD02 (cont)

Q	Solution	Marks	Total	Comments
2(a)	<i>SAET</i> has maximum day journey of 9 hrs whereas for <i>SADT</i> max day journey is 10 hrs	M1	2	Reasonable understanding
		A1		with 9 and 10 specifically mentioned
(b)	Stage Initial State Action Value	M1	8	General idea of stage and state
	1 <i>D</i> <i>DT</i> 5*	A1		First stage correct (may be reversed)
	<i>E</i> <i>ET</i> 7*			
	2 <i>A</i> <i>AD</i> $\max(10,5) = 10$ <i>AE</i> $\max(9,7) = 9^*$	M1		Idea of minimax One pair of actions correct
		A1		
	<i>B</i> <i>BD</i> $\max(9,5) = 9$ <i>BE</i> $\max(8,7) = 8^*$	A1		All values in second stage correct
	3 <i>S</i> <i>SA</i> $\max(7,9) = 9$ <i>SB</i> $\max(8,8) = 8^*$ <i>SC</i> $\max(9,10) = 10$	A1		All values correct at all 3 stages
		M1		
		A1		
Working back along * values to find Minimax route is <i>SBET</i>				Complete/enumeration or network with each stage and state carefully described if no evidence of minimax Maximum mark M1, A1 Minimax route <i>SBET</i> marks may also be earned if not finding minimum time through the network. M1 A1
Total			10	

MD02 (cont)

Q	Solution	Marks	Total	Comments
3				
(a)	Network	M1 A3	4	SCA -1 ee
(b)	Forward pass All correct	M1 A1	2	
(c)	Backward pass All correct	M1 A1	2	
(d)(i)	Project completion time 36 hours	B1✓	1	
(ii)	Critical path <i>BCEGHJ</i> Earliest start + activity duration = latest finish time	M1 A1 E1	3	SCA All correct
(e)(i)	<i>I</i> now has new earliest time 29+3 = 32	M1 A1	2	Extra 3 hours on edge <i>HI</i> or new activity between <i>H</i> and <i>I</i> of duration 3
(ii)	<i>I</i> now becomes critical and increases <i>J</i> earliest start time to 35 New completion time is 37 hours	M1 A1	2	
	Total		16	

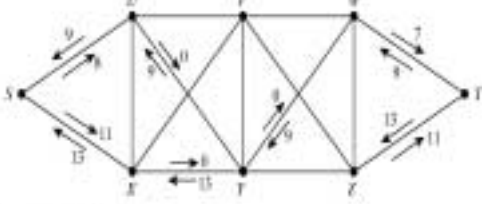
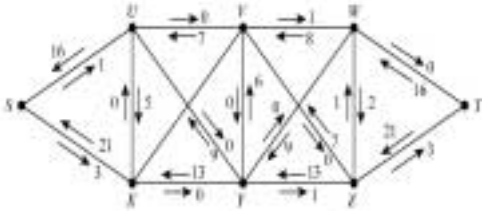
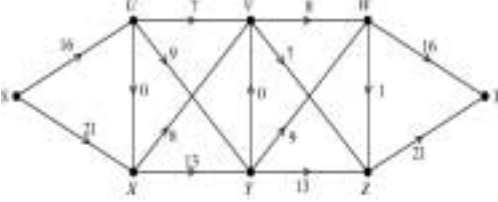
MD02 (Cont)

Q	Solution	Marks	Total	Comments																																																															
4(a)	$4x + 5y \leq 36$	M1	2	SCA at LHS and RHS																																																															
	$2x + y \leq 12$ $5x + 2y \leq 35$	A1		All correct with correct inequalities																																																															
(b)(i)	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>r</i></th> <th><i>s</i></th> <th><i>t</i></th> <th><i>value</i></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-3</td> <td>-2</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>4</td> <td>5</td> <td>1</td> <td>0</td> <td>0</td> <td>36</td> </tr> <tr> <td>0</td> <td>1*</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>6</td> </tr> <tr> <td>0</td> <td>5</td> <td>2</td> <td>0</td> <td>0</td> <td>1</td> <td>35</td> </tr> <tr> <td>1</td> <td>0</td> <td>$-\frac{1}{2}$</td> <td>0</td> <td>$\frac{3}{2}$</td> <td>0</td> <td>18</td> </tr> <tr> <td>0</td> <td>0</td> <td>3*</td> <td>1</td> <td>-2</td> <td>0</td> <td>12</td> </tr> <tr> <td>0</td> <td>1</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>6</td> </tr> <tr> <td>0</td> <td>0</td> <td>$-\frac{1}{2}$</td> <td>0</td> <td>$-\frac{5}{2}$</td> <td>1</td> <td>5</td> </tr> </tbody> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>r</i>	<i>s</i>	<i>t</i>	<i>value</i>	1	-3	-2	0	0	0	0	0	4	5	1	0	0	36	0	1*	$\frac{1}{2}$	0	$\frac{1}{2}$	0	6	0	5	2	0	0	1	35	1	0	$-\frac{1}{2}$	0	$\frac{3}{2}$	0	18	0	0	3*	1	-2	0	12	0	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	6	0	0	$-\frac{1}{2}$	0	$-\frac{5}{2}$	1	5	M1		Identifying pivot and possibly dividing by 2
	<i>P</i>	<i>x</i>	<i>y</i>	<i>r</i>	<i>s</i>	<i>t</i>	<i>value</i>																																																												
	1	-3	-2	0	0	0	0																																																												
	0	4	5	1	0	0	36																																																												
	0	1*	$\frac{1}{2}$	0	$\frac{1}{2}$	0	6																																																												
	0	5	2	0	0	1	35																																																												
	1	0	$-\frac{1}{2}$	0	$\frac{3}{2}$	0	18																																																												
	0	0	3*	1	-2	0	12																																																												
	0	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	6																																																												
	0	0	$-\frac{1}{2}$	0	$-\frac{5}{2}$	1	5																																																												
			m1		Row operations																																																														
			A1		Correct tableau																																																														
	Next <i>y</i> pivot on 3	M1																																																																	
	<table border="1" style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td>1</td> <td>0</td> <td>0</td> <td>$\frac{1}{6}$</td> <td>$\frac{7}{6}$</td> <td>0</td> <td>20</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>$\frac{1}{3}$</td> <td>$-\frac{2}{3}$</td> <td>0</td> <td>4</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td>$-\frac{1}{6}$</td> <td>$\frac{5}{6}$</td> <td>0</td> <td>4</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>$\frac{1}{6}$</td> <td>$-\frac{17}{6}$</td> <td>1</td> <td>7</td> </tr> </tbody> </table>	1	0	0	$\frac{1}{6}$	$\frac{7}{6}$	0	20	0	0	1	$\frac{1}{3}$	$-\frac{2}{3}$	0	4	0	1	0	$-\frac{1}{6}$	$\frac{5}{6}$	0	4	0	0	0	$\frac{1}{6}$	$-\frac{17}{6}$	1	7	m1		Row operations																																			
1	0	0	$\frac{1}{6}$	$\frac{7}{6}$	0	20																																																													
0	0	1	$\frac{1}{3}$	$-\frac{2}{3}$	0	4																																																													
0	1	0	$-\frac{1}{6}$	$\frac{5}{6}$	0	4																																																													
0	0	0	$\frac{1}{6}$	$-\frac{17}{6}$	1	7																																																													
		A1		Correct tableau																																																															
	Optimal since no negative numbers in top row	B1	7																																																																
(ii)	$P = 20$ $x = 4, y = 4$	B1✓ B1✓	2	FT ONLY if no negs in top row																																																															
(iii)	$r = 0, s = 0, t = 7$ at optimum	B1✓	1																																																																
Total			12																																																																

MD02 (cont)

Q	Solution	Marks	Total	Comments																				
<p>5(a)</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td>4</td> <td>3</td> <td>5</td> <td>Min</td> </tr> <tr> <td></td> <td>-1</td> <td>5</td> <td>-2</td> <td>3*</td> </tr> <tr> <td></td> <td>1</td> <td>2</td> <td>3</td> <td>-2</td> </tr> <tr> <td>Max</td> <td>4*</td> <td>5</td> <td>5</td> <td>1</td> </tr> </table> <p>Since $3 \neq 4 \Rightarrow$ no stable solution</p>		4	3	5	Min		-1	5	-2	3*		1	2	3	-2	Max	4*	5	5	1	<p>M1 A1 A1</p>	<p> 3</p>	<p>Either marginal row or column all values correct</p>
	4	3	5	Min																				
	-1	5	-2	3*																				
	1	2	3	-2																				
Max	4*	5	5	1																				
<p>(b)</p>	<p>P_1 dominates P_3 ; $(1,2,3) < (4,3,5)$</p> <p>So it is unwise to play P_3</p>	<p>E1</p>	<p>1</p>																					
<p>(c)</p>	<p>P chooses P_1 with probability p So chooses P_2 with probability $1 - p$</p>																							
<p>Expected gains when Q plays</p>	<p>$Q_1 : 4p - (1-p) = 5p - 1$</p>	<p>M1</p>		<p>Attempt at at least 2</p>																				
<p>$Q_2 : 3p + 5(1-p) = 5 - 2p$</p>		<p>A1</p>		<p>All 3 correct (simplified)</p>																				
<p>$Q_3 : 5p - 2(1-p) = 7p - 2$</p>		<p>A1</p>																						
<p>Plot expected gains against p for $0 \leq p \leq 1$</p>		<p>M1</p>																						
<p>Choose highest point of region below lines</p>	$5p - 1 = 5 - 2p$	<p>M1</p>																						
<p>Therefore P plays P_1 with probability $\frac{6}{7}$</p>	<p>leading to $p = \frac{6}{7}$</p>	<p>A1</p>																						
<p>and plays P_2 with probability $\frac{1}{7}$</p>		<p>B1✓</p>	<p>7</p>																					
Total			<p>11</p>																					

MD02 (cont)

Q	Solution	Marks	Total	Comments												
6(a)(i)	9+7+0+9+13=38	B1	1	< their value of cut ≤ 38 M0 for “equals” their cut												
	(ii) Maximum flow is less than or equal to 38	M1 A1✓	2													
(b)	<i>SUYWT</i> flow of 9	B1	2													
	<i>SXYZT</i> flow of 13	B1														
(c)(i)	Indicating flows from (b) on network	M1		Preferably as backward flows												
	 <p>with augmentation leading to something such as</p>  <table border="1" data-bbox="204 1077 703 1294"> <thead> <tr> <th>Route</th> <th>Flow</th> </tr> </thead> <tbody> <tr> <td><i>SUYWT</i></td> <td>9</td> </tr> <tr> <td><i>SXYZT</i></td> <td>13</td> </tr> <tr> <td><i>SUVWT</i></td> <td>7</td> </tr> <tr> <td><i>SXVZT</i></td> <td>7</td> </tr> <tr> <td><i>SXVWZT</i></td> <td>1</td> </tr> </tbody> </table> <p>Flow augmentation (many possibilities)</p> <p style="text-align: right;"> <i>SUVWT</i> 7 <i>SXVZT</i> 7 <i>SXVWZT</i> 1 </p>	Route	Flow	<i>SUYWT</i>	9	<i>SXYZT</i>	13	<i>SUVWT</i>	7	<i>SXVZT</i>	7	<i>SXVWZT</i>	1	M1A1 m1A1 A1	6	
Route	Flow															
<i>SUYWT</i>	9															
<i>SXYZT</i>	13															
<i>SUVWT</i>	7															
<i>SXVZT</i>	7															
<i>SXVWZT</i>	1															
(ii)	Network showing maximum flow Several possibilities	B1														
	 <p>Maximum flow is 37</p>	B1	2	Or {S, U, X / V, W, Y, Z, T}												
(iii)	Attempt to find cut through saturated arcs Cut through <i>UV, UY, XV, XY</i>	M1 A1	2													
	Total		15													
	Total		75													